MATH 104
Solution
Solving triangles

This problem concerns Handout 7, Exercise 5.

The given information is $R = 10$, $b = 7$, and $C = \frac{\pi}{4}$. To solve the triangle, we need to find a, c, A, and B. Knowing the area of the triangle allows us to use the general formula

$$\begin{aligned} R &= \frac{1}{2}ab\sin C \Rightarrow \\ 10 &= \frac{1}{2}a\,(7)\sin\frac{\pi}{4} \Rightarrow \\ a &\approx 4.0406. \end{aligned}$$

Then, via the law of cosines, we can find side c :

$$\begin{aligned} c^2 &= 4.0406^2 + 7^2 - 2\,(4.0406)\,(7)\cos\frac{\pi}{4} \Rightarrow \\ c &\approx 5.0325. \end{aligned}$$

Using the law of sines, we can now solve for angle A or B. We choose the latter:

$$\begin{aligned} \frac{\sin\frac{\pi}{4}}{5.0325} &= \frac{\sin B}{7} \Rightarrow \\ \sin B &\approx .9836 \Rightarrow \\ B &\approx 1.3892. \end{aligned} \tag{1}$$

At this point in the solution process, possibility (1) leads to the following conclusion.

$$B \approx 1.3892 \Rightarrow A \approx \pi - \frac{\pi}{4} - 1.3892 \approx .9670.$$

Note that this means that

$$B \approx 1.3892 > A \approx .9670 > C = \frac{\pi}{4} \approx .7854.$$

Then from geometry, we know this implies that

$$b > a > c.$$

However, from previous computations above, we have that $a < c$. Then possibility (1) must be invalid. Having eliminated B as a first quadrant (acute) angle, we instead consider the possibility that its terminal side lies in the second quadrant (obtuse):

$$\begin{aligned} \sin B &\approx .9836 \Rightarrow \\ B &\approx \pi - 1.3892 = 1.7542. \end{aligned} \tag{2}$$

Then

$$A \approx \pi - \frac{\pi}{4} - 1.7542 \approx .6038.$$

This time we have that

$$B > C > A,$$

which coincides with our previous computations indicating

$$b > c > a.$$

Then we get a unique solution provided by possibility (2) only.

$$A \approx .6038, \quad B \approx 1.7542, \quad a \approx 4.0406, \quad c \approx 5.0325.$$